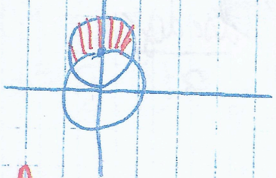
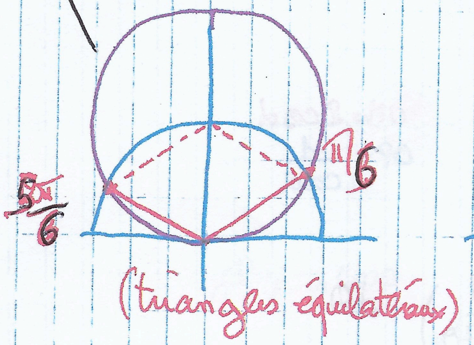
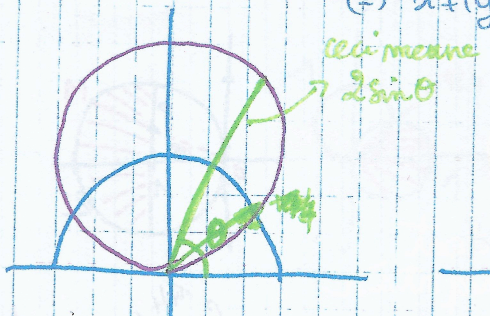
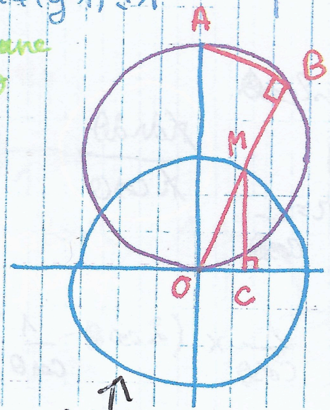
autre méthode $\begin{cases} x^2+y^2=1 \\ x^2+y^2-2y=0 \end{cases} \Leftrightarrow \begin{cases} y=1/2 \\ x=\pm\sqrt{3}/2 \end{cases}$ d'où $\theta = \pi/6$ ou $5\pi/6$



④ $\displaystyle\iint \frac{dx\,dy}{\sqrt{4-x^2-y^2}}$ $\quad D \begin{cases} x^2+y^2 \geq 1 \\ x^2+y^2-2y \leq 0 \end{cases} \Leftrightarrow x^2+(y-1)^2 \leq 1$







OMC et OAB ont les mêmes angles. Or, OA=2OM. C'est pourquoi OB=2×MC

ou choisir

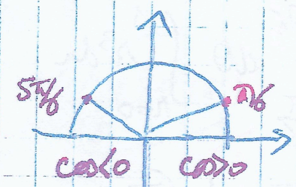
autre méthode $\begin{cases} r^2 \geq 1 \\ r^2-2r\sin\theta \leq 0 \end{cases} \Leftrightarrow \begin{cases} r \geq 1 \\ r \leq 2\sin\theta \end{cases}$

$$I = \int_{\theta=\pi/6}^{5\pi/6} \int_{r=1}^{2\sin\theta} \frac{r\,dr\,d\theta}{\sqrt{4-r^2}}$$

$$I = \int_{\theta=\pi/6}^{5\pi/6} \left[-\sqrt{4-r^2}\right]_{r=1}^{2\sin\theta} d\theta$$

$$I = \int_{\theta=\pi/6}^{5\pi/6} \left(-2\sqrt{1-\sin^2\theta}+\sqrt{3}\right) d\theta$$

$$I = \int_{\pi/6}^{5\pi/6} -2|\cos\theta|\,d\theta + \frac{2\pi}{3}\sqrt{3}$$



$$I = -4\int_{\pi/6}^{\pi/2} \cos\theta\,d\theta + \frac{2\pi\sqrt{3}}{3}$$

$$I = -4\left[\sin\theta\right]_{\pi/6}^{\pi/2} + \frac{2\pi\sqrt{3}}{3}$$

$$I = -4\left(1-\frac{1}{2}\right) + \frac{2\pi\sqrt{3}}{3} = 2\left(\frac{\pi}{\sqrt{3}}-1\right)$$

le ③ page suivante